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$\therefore u=\frac{4395}{59319}$, $v=\frac{4519}{59319}$, $x=\frac{4737}{59319}$, $y=\frac{6122}{59319}$.

Let $a=4$, $b=7$, $c=8$, $d=17$, $h=18$.

$\therefore u=\frac{11728}{157464}$, $v=\frac{12007}{157464}$, $x=\frac{12176}{157464}$, $y=\frac{16577}{157464}$.

Other values can be found for u, v, x, y.

77. **Proposed by JOHN M. COLAW, A. M., Monterey, Va.**

Find (1) three consecutive numbers whose sum is a cube, and (2) three consecutive numbers the sum of whose cubes is a cube.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

(1). Let $n-1$, n, and $n+1$ be any three consecutive numbers.

Then $(n-1)+n+(n+1)=3n=$ a cube $=27m^3$.

Whence $n=9m^3$.

$\therefore 9m^3-1$, $9m^3$, and $9m^3+1$ are the general expressions for three consecutive numbers whose sum is a cube.

Take $m=1$; then $8+9+10=27=3^3$.

Take $m=2$; then $71+72+73=216=6^3$; etc.

(2). $(n-1)^3+n^3+(n+1)^3=3n^3+6n=$ a cube $=27m^3$.

Whence $n^3+2n=9m^3$.

Put $m=an$; then $n^3+2n=9a^3n^3$.

Whence $n^2+2=9a^3n^2$; and $n^2=2/(9a^3-1)$.

To obtain n integral, a must be fractional.

Put $a=1/b$; then $n^2=2b^3/(9-b^3)$.

To avoid imaginary results, $b<2\frac{1}{12}$.

The only integral values that can be assigned to b are 1 and 2.

Take $b=1$; then $n=\frac{1}{2}$.

Whence $(-\frac{1}{2})^3+(\frac{1}{2})^3+(\frac{3}{2})^3=(\frac{3}{2})^3$.

Take $b=2$; then $n=4$.

Whence $3^3+4^3+5^3=6^3$.

This is the only set of three consecutive integers the sum of whose cubes is a cube.

Fractional values of b give fractional values for n.

When $b=0$, $n=0$.

Whence $(-1)^3+0^3+1^3=0^3$.

Also solved by *CHARLES C. CROSS, JOSIAH H. DRUMMOND, ALOIS F. KOVARIK, NELSON L. RORAY, J. SCHEFFER, ELMER SCHUYLER*, and *G. B. M. ZERR.*

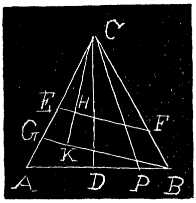
AVERAGE AND PROBABILITY.

81. **Proposed by LON C. WALKER, Assistant in Mathematics, Leland Stanford, Jr., University, Palo Alto, Cal.**

Find (1) the mean distance of all points on a side of an equilateral triangle from the opposite vertex; and (2), the average length of a line drawn at random across an equilateral triangle.

I. Solution by the PROPOSER.

1. Let each side$=a$ of the equilateral triangle ABC delineated in the annexed figure. Then its altitude $CD=\frac{1}{2}a\sqrt{3}$.



Put $DP=x$, then $CP=\sqrt{(x^2+\frac{3}{4}a^2)}$.
Hence the required mean distance is

$$M=\frac{\int_0^{\frac{1}{2}a}\sqrt{(x^2+\frac{3}{4}a^2)}dx}{\int_0^{\frac{1}{2}a}dx}=\tfrac{1}{4}a(2+3\log\sqrt{3}).$$

2. Suppose EF the random line. Draw GB parallel to EF, and CHK perpendicular to BG. Put $CH=x$ and $\angle CEF=\theta$. Then

$$EF=\frac{x\sin\frac{1}{3}\pi}{\sin\theta\sin(\frac{1}{3}\pi+\theta)}.$$

$CK=a\sin\theta$, and the required average length is

$$L=\frac{\int_0^{\frac{1}{3}\pi}\int_0^{a\sin\theta}x\sin\frac{1}{3}\pi\,\mathrm{cosec}\theta\,\mathrm{cosec}(\frac{1}{3}\pi+\theta)d\theta dx}{\int_0^{\frac{1}{3}\pi}\int_0^{a\sin\theta}d\theta dx}$$

$$=a\int_0^{\frac{1}{3}\pi}\sin\tfrac{1}{3}\pi\sin\theta\,\mathrm{cosec}(\tfrac{1}{3}\pi+\theta)d\theta=\tfrac{1}{4}a\int_0^{\frac{1}{3}\pi}[\sqrt{3}-3\cot(\tfrac{1}{3}\pi+\theta)]d\theta=\tfrac{1}{12}\pi a\sqrt{3}.$$

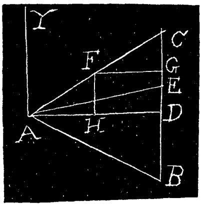
II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

(1). Let $AC=2a$, $DE=x$. Then $AD=a\sqrt{3}$, $AE=\sqrt{(3a^2+x^2)}$.

$$\therefore M=\frac{\int_0^a\sqrt{(3a^2+x^2)}dx}{\int_0^a dx}=\frac{1}{a}\int_0^a\sqrt{(3a^2+x^2}dx.$$

$\therefore M=\frac{1}{4}a(4+3\log 3)$.

If we regard $\angle DAE=\theta$ as the variable,



$$M=a\sqrt{3}\frac{\int_0^{\frac{1}{6}\pi}\sec\theta d\theta}{\int_0^{\frac{1}{6}\pi}d\theta}=\frac{6a\sqrt{3}}{\pi}\int_0^{\frac{1}{6}\pi}\sec\theta d\theta.$$

$$\therefore M=\frac{3a\sqrt{3}\,\log 3}{\pi}.$$

(2). Let FG be the random line, $AH=x$, $DG=y$.
Then $FH=\frac{1}{3}\sqrt{3}x$, $FG=\sqrt{[(a\sqrt{3}-x)^2+(y-\frac{1}{3}\sqrt{3}x)^2}=l$.

$$\triangle=\text{average length}=\frac{\int_0^{a\sqrt{3}}\int_0^a l\,dx\,dy}{\int_0^{a\sqrt{3}}\int_0^a dx\,dy}$$

$$\triangle=\frac{1}{a^2\sqrt{3}}\int_0^{a\sqrt{3}}\int_0^a\sqrt{[(a\sqrt{3}-x)^2+(y-\tfrac{1}{3}\sqrt{3}x)^2]}\,dx\,dy$$

$$=\frac{1}{12a^2\sqrt{3}}\int_0^{a\sqrt{3}}\Big[4(a\sqrt{3}-x)^2+x\sqrt{[9a^2+(4x-3\sqrt{3}\,a)^2]}$$

$$-6(a\sqrt{3}-x)^2\log\left(\frac{\sqrt{[9a^2+(4x-3\sqrt{3}\,a)^2]}-2x}{6(\sqrt{3}-x)}\right)\Big]dx.$$

$\therefore\ \triangle=(a/16)[6+2\sqrt{3}+4\log3+\log(3+2\sqrt{3})]$.

82. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Find the average area of the quadrilateral formed by joining the extremities of two chords perpendicular to each other and passing through a point at a distance a from the center of a circle radius R.

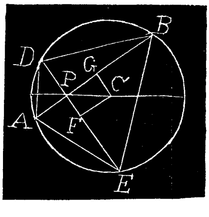
Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; and the PROPOSER.

Let $\triangle$=average area, $\angle BPC=\theta$, P the given point, C the center of the cirle, $CP=a$, $CB=R$.

CASE I. $a<R$, $CG=a\sin\theta$, $CF=a\cos\theta$.

$\therefore\ AB=2\sqrt{(R^2-a^2\sin^2\theta)}$, $DE=2\sqrt{(R^2-a^2\cos^2\theta)}$.

Area $ADBE=2\sqrt{[(R^2-a^2\sin^2\theta)(R^2-a^2\cos^2\theta)]}$
$=2R^2\sqrt{[(1-e^2\sin^2\theta)(1-e^2\cos^2\theta)]}$ where $e^2=a^2/R^2$.



$$\therefore\ \triangle=2R^2\int_0^{\frac{1}{2}\pi}\sqrt{[(1-e^2\sin^2\theta)(1-e^2\cos^2\theta)]}\,d\theta\Big/\int_0^{\frac{1}{2}\pi}d\theta$$

$$=\frac{2R^2}{\pi}(2-e^2)\,E\left(\frac{e^2}{2-e^2},\ \tfrac{1}{2}\pi\right).$$

(See page 356, No. 10, Vol. I, for above integration.

CASE II. $a>R$ and $a<R\sqrt{2}$, $PF=a\cos\theta$, $PG=a\sin\theta$:
$PB=a\cos\theta+\sqrt{(R^2-a^2\sin^2\theta)}$, $PA=a\cos\theta-\sqrt{(R^2-a^2\sin^2\theta)}$.
$PE=a\sin\theta+\sqrt{(R^2-a^2\sin^2\theta)}$, $PD=a\sin\theta-\sqrt{(R^2-a^2\cos^2\theta)}$.
Area $ABED=\frac{1}{2}(BP.PE--AP.PD)$

$$=a[\sqrt{(R^2-a^2\sin^2\theta)}\sin\theta+\sqrt{(R^2-a^2\cos^2\theta)}\cos\theta].$$

The limits of θ are $\frac{1}{2}\pi-\sin^{-1}(R/a)=\theta''$ and $\sin^{-1}(R/a)=\theta'$.